SELECTED TOPICS IN MODERN GEOMETRY
MA 624

Course Description

This course is prefaced by a careful examination of the foundations of geometry. Major topics include: finite geometry, synthetic and coordinate projected geometry, hyperbolic geometry, elliptic geometry, differential geometry, and topology. Considerable attention is given to the modern alliance of geometry with linear and abstract algebra.

Goals of the Course

1. To enrich the students appreciation of the geometries used in the modern world of mathematics by studying elementary and advanced Euclidean geometry and then continuing the investigation by studying the non-Euclidean geometries of Riemann and Lobachevski.
2. To give students an appreciation of the evolution of geometry from the five postulates of Euclid to a hyperbolic description of geometry.
3. To give students rigorous practice in constructing geometric proofs,
4. To encourage students who will be teaching geometry to begin thinking about how they will inspire their students with this material.

Instructional Procedures

1. Lecture/Discussion
2. Reading and Problems
3. Assigned research project. The topic will be chosen by the students and approved by the instructor. It will be on an advanced topic in geometry.

Course Content

1. Introduction
2. Incident Geometry in Planes and Spaces
3. Distance and Congruence
   A. Function and the set-theoretic interpretation of functions and relations.
   B. Distance Function
   C. Betweeness
   D. Segments, rays, angles, and triangles
   E. Congruence of segments

4. Separation in Planes and Spaces
   - A. Convexity and Separation
   - B. Incidence theorems
   - C. Convex quadrilaterals
   - D. Separation of space by Planes
   - E. Seven bridges of Konigsbreg
5. Angular Measures
6. Congruence between triangles
   - A. Basic theorems and terminology
   - B. Independence of SAS Postulate
   - C. Existence of perpendiculars
7. Absolute Plane Geometry
   - A. Polygonal inequality
   - B. Saccheri Quadrilaterals
   - C. Basic inequality for angle-sums in a triangle
8. Parallel Projections
   - A. Uniqueness of Parallels
   - B. Parallel Projection
9. Hyperbolic Geometry
   - A. Closed triangles and sums
   - B. Defect of a triangle
   - C. Bolyai's Theorem
   - D. Impossibility of Euclidean area-theory in Hyperbolic Geometry
   - E. Uniqueness of Hyperbolic Area
   - F. Alternate forms of Parallel Postulate
10. Consistency of the Hyperbolic Postulates
    - A. Inversion of a Plane
    - B. Preservation of angular measures under inversion
11. Special Topics in Riemannian Geometry

Evaluation Measures
1. Three hourly exams	50%	
2. Comprehensive final exam	25%	
3. Research project	25%	

Bibliography

Required Text
Moise, Edwin E., Elementary Geometry from an Advanced Standpoint, 3rd Ed., Addison-Wesley Pub. Co., NY, 1990.

Supporting Bibliography

Adler, Claire Fisher, Modern Geometry as an Integrated First Course, 2nd Ed., McGraw-Hill Pub., NY, 1967.

Clemens, C. Herbert & Michael Clemens, Geometry for the Classroom, Springer-Verlag, NY, 1991.

Coexter, H.S.M., Introduction to Geometry, 2nd Ed., John Wiley & Sons, NY, 1969.

Geltner, Peter B., & Darrell J. Peterson, Geometry for College Students, 3rd Ed., PWS Pub., CO, 1995.

Greenberg, Marvin, Euclidean and Non-Euclidean Geometries, 3rd Ed., W.H. Freeman & Co., NY, 1993.

Jacobs, Harold, Geometry, 2nd Ed., W.H. Freeman & Co., NY, 1987.

Rich, Barnett, Theory and Problems of Geometry, Schaum's Outline Series, McGram-Hill, NY, 1989.

Ryan, Patrick, Euclidean and Non-Euclidean Geometry: An Analytic Approach, Cambridge University Press, NY, 1991.

Smart, James, Modern Geometries, 4th Ed., Brooks/Cole, Pacific Grove, CA, 1994.